Compressor Noise Generated by Fluctuating Lift Resulting from Rotor-Stator Interaction

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It is well known that the blades of an axial-flow compressor do not operate in uniform conditions but are subject to variations in incident flow which are due to the presence of other blade rows in the compressor. As a result the lift force exerted by the fluid on the blade will also vary and be a source of dipole noise. The purpose of this investigation is to estimate the intensity of noise generated in this manner by a compressor stage in a gas turbine.

Kemp[1] has developed the theory for the determination of the lift of an airfoil in unsteady flow, and Kemp and Sears[2] have applied this method to a compressor stage. This note extends the results of Ref. 2 to estimate the intensity of sound that is generated by the stage due to fluctuating lift forces. Experimental results for a blade in cascade producing substantial wakes which excite the following row are also treated by the method of Ref. 1, and a further estimate of the sound generated is made.

The relative upwash on a thin stationary aerofoil lying along the x axis is given by

$$v(x,t) = v_0 e^{i\nu t} e^{-i\mu x/V}$$

where ν and μ are constants. μ may be real or complex; the imaginary part of μ allows for the attenuation of v with varying x. The lift per unit span is the sum of the steady-state contribution L_0 and the time-dependent part $L(t)$. Kemp[1] has shown that

$$L(t) = 2\pi\rho c\ V v_0 K_L(\omega, \lambda) e^{i\nu t}$$

where V = flow velocity in the x-direction and c is the blade semichord. $\omega = \nu c/V$ and $\lambda = \mu c/V$ are nondimensional parameters and,

$$K_L(\omega,\lambda) = J(\lambda)C(\omega) + (i\omega\ J(\lambda)/\lambda)$$

with

$$J(\lambda) = J_0(\lambda) - i\,J_1(\lambda)$$

$$C(\omega) = K_1\ (i\omega)/[K_0(i\omega) + K_1(i\omega)]$$

The above results apply to a thin airfoil at small incidence in an incompressible fluid.

The fluctuating lift may now be regarded as an element of a line dipole of strength A, where $A = 2\pi\rho c\ V v_0 |K_L|$. The total power radiated per unit span is $P = \pi\nu A^2/2\rho a^2$, where a is the velocity of sound. To find the total power radiated by a blade row it remains to sum P from root to tip over all blades.

The nonuniformity of the flow incident on a blade results from two effects: (1) variations in the potential flow due to the presence of other blade rows in the flow, and (2) the wakes of the upstream blade row. Kemp and Sears[2] have worked a typical example for a single compressor stage for the case of variations in potential flow. A 50 percent-reaction stage was used with either flat-plate blading or blading with circular-arc camber. Equal rotor and stator spacings were chosen with $2c/S = 1$, where S is the blade pitch. It is of interest to interpret their results for $L(t)$ in terms of noise generation and examine the effect of axial spacing between rotor and stator for this case.

In a turbojet, at full power, jet noise is dominant, but at lower exit velocities other sources of noise provide a substantial contribution to the total noise output. For this reason we consider a compressor operating in a turbojet at 30 percent thrust when the following typical first-stage values may be assigned in order to calculate the noise generated: blade speed at a mean radius = 500 ft/sec; axial flow velocity at a mean radius = 350 ft/sec; blade lift coefficient = 0.8 (corresponding to $7^1/_2°$ incidence on a flat plate); number of blades = 31; blade root/blade tip = 0.3; rotational speed = 3,000 rpm; height of blades = 10 in. The total radiated noise per unit intake area is plotted in Fig. 1 against axial separation b, for both upstream and downstream rows for both types of blading.

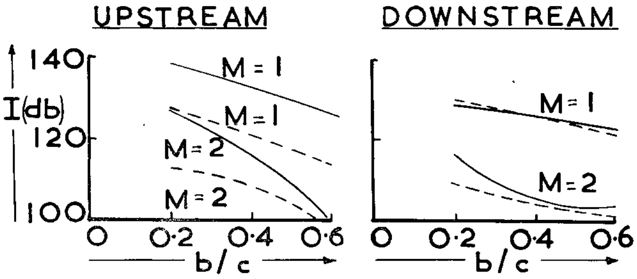

Fig. 1 Intensity of sound per unit duct area, I, as a function of axial spacing b. Dash curves; circular-arc cambered upstream and downstream blades. Solid curves; circular-arc cambered upstream blades and flat-plate downstream blades. m = 1 first (fundamental) harmonic, m = 2 second harmonic

In Ref. 3 the method of Ref. 2 has been extended to predict the fluctuating lift on a blade row which has been subject to the wakes of the preceding row. The work again is applicable to thin airfoil at small incidence, and it was found that in these circumstances the fluctuating lift was of the same order of magnitude as that produced by variation in the potential flow. In practice where the compressor is working away from its design point the incident flow angle is not small and substantial wakes result. Fig. 2 shows an experimental velocity traverse of the flow incident on a first-stage rotor near the stalling condition. The strong wake is clearly seen. The axial spacing parameter $b/c = 0.5$ in this case. A Fourier analysis has been made of this curve and the fluctuating-lift noise calculated on the assumption that the velocity variations of Fig. 2 pertain over the whole of the blade. The results are given in Table 1.

Table 1

Harmonic	1	2	3	4	5
Sound Intensity/Duct Area (db)	124	134	125	124	117

The note has shown that rotor-stator interaction is an important source of noise. It has been confined to estimating

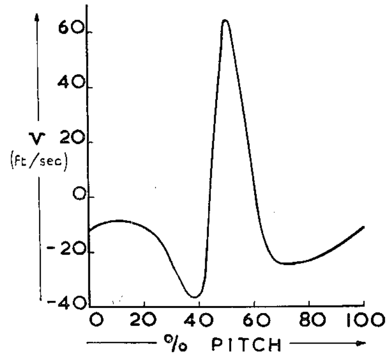

Fig. 2 Upwash velocity

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the intensity of noise generated by this mechanism and has not attempted to solve the problem of the transmission of the noise out of the inlet duct, which is of equal importance. It has been shown in the case of potential-flow variations that the upstream row generated as much noise as the following row. As the leading row in a compressor is in a preferential position from the propagation point of view, this may be the major source of noise heard outside the inlet duct. When wakes become appreciable at high incidence the total amount of noise generated is increased though the fundamental is of the same order of magnitude as in the low-incidence case. The sharp wake produces substantial increases in the high harmonics, and in the case worked the second harmonic dominated by 10 db.

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On the Coupling between Orthogonal Couette and Pressure Flows

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In the problem of leakage around high-speed shafts and also short journal bearings, a weak pressure flow acts at right angles to a strong Couette flow. For the case where the motion is turbulent, Tao[1] has been able to solve the short-bearing problem by neglecting any coupling effects between the two component flows. Constantinescu[2] has written mixing-length equations for such flows, also assuming the components to be independent. On the other hand, Prandtl[3] provided a mixing-length equation which permits accounting for coupling effects. The purpose of this note is to see what this predicts and to examine the restrictions involved in its application.

The mixing-length equation may be written in Cartesian coordinates as

$$T_{xy} = (\rho l^2 J + \mu)[(\partial u/\partial y) + (\partial v/\partial x)] \qquad (1)$$

where J is defined by

$$J^2 = 2[(\partial u/\partial x)^2 + (\partial v/\partial y)^2 + (\partial w/\partial z)^2] + [(\partial w/\partial y) + (\partial v/\partial z)]^2 + [(\partial u/\partial z) + (\partial w/\partial x)]^2 + [(\partial v/\partial x) + (\partial u/\partial y)]^2 \qquad (2)$$

Simplifying to apply to parallel flow (which is uniform in the sense that $\partial u/\partial x = \partial w/\partial y = \partial v/\partial z \equiv 0$), this becomes

$$T_{xy} = [\rho l^2 \sqrt{(\partial u/\partial y)^2 + (\partial w/\partial z)^2} + \mu](\partial u/\partial y) \qquad (3)$$

Interchanging coordinates,

$$T_{zy} = [\rho l^2 \sqrt{(\partial w/\partial z)^2 + (\partial u/\partial y)^2} + \mu](\partial w/\partial y) \qquad (4)$$

Here the x and z coordinates are parallel to the boundary surfaces, and the y coordinate is normal to these. The velocity u in the x-direction represents the Couette flow, and the velocity w in the z-direction represents the pressure flow. The fluid density is ρ, the viscosity is μ, and the mixing length is l.

When τ_{zy} is the wall stress of the pressure flow, it is possible to write

$$\tau_{zy}(1 - y/b) = \rho[l^2 + \Delta(l^2)] \times \{(\partial u/\partial y)^2[(\partial w/\partial y)/(\partial u/\partial z)] + \mu(\partial u/\partial y)[(\partial w/\partial y)/(\partial u/\partial y)]\} \qquad (5)$$

If it is assumed that $\Delta(l^2)/l^2$ is of the same order or smaller than w/u, and if $w/u \ll 1$, this simplifies to

$$\tau_{zy}(\partial u/\partial y)[1 - (y/b)] = \tau_{zy} w \qquad (6)$$

where b is half the distance separating the plates. To permit integration, a Couette-flow profile must be used. Since it has been[4] shown that the profile $u/u_b = (y/b)^{1/7}$ fits experimental data, and since small departures are smoothed out by the integration process, Eq. (6) may be reduced to

$$T_{zy} u[1 - (1/8)(y/b)] = T_{xy} w \qquad (7)$$

Applying this at a point halfway between plates where $y = b$, this becomes

$$T_{zy}/T_{xy} = (8/7)(w_b/u_b) \qquad (8)$$

It is of interest that, if the z-direction flow had been an incremental Couette flow, the quantity 8/7 in Eq. (7) would be replaced by 1. Thus, it is seen that the resistance relative to a given wall is not greatly affected by the two types of pressure distribution.

Quite aside from questions concerning the validity of Eqs. (1) and (2), note that: (a) The solution did not require that viscous effects be neglected. (b) It was not necessary to assume an equation for mixing-length variation. (c) It was not necessary to assume that the mixing-length function remains constant, so long as the order of $\Delta(l^2)/l^2$ is the same or smaller—i.e., higher-order—than w/u, for the case where $w/u \ll 1$. As simplified in Eq. (4), the basic equation used is equivalent to stating that the eddy viscosity of the primary flow also serves as the eddy viscosity of the incremental flow. Furthermore, in Eq. (4) it is seen that the eddy viscosity ϵ is determined by the vector derivative of the combined velocity with respect to y; hence,

$$\epsilon = \rho l^2 \sqrt{2(\partial u/\partial y)^2 + (\partial w/\partial z)^2} \qquad (9)$$

As used, however, it has not been necessary to accept this relationship wholly. One need only accept that, in whatever true equation governs, the relative participation of the $\partial w/\partial y$ is such that it tends to vanish as $\partial w/\partial y \to 0$.

On the basis of the above arguments, one is tempted to conclude that for Eq. (8) to be a valid approximation, only a very low degree of validity is required of the equations which went into its derivation.

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